USN ☐☐☐☐☐☐☐☐☐☐   **15MATDIP31**

Third Semester B.E. Degree Examination, Aug./Sept.2020
Additional Mathematics – I

Time: 3 hrs.   Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1   a.   Express $\dfrac{5+2i}{5-2i}$ in the form $xi + iy$.   **(06 Marks)**

    b.   Find the modulus and amplitude of $\dfrac{(1+i)^2}{3+i}$ **(05 Marks)**

    c.   If $\vec{a} = (3, -1, 4)$, $\vec{b} = (1, 2, 3)$, $\vec{c} = (4, 2, -1)$ find $\vec{a} \times (\vec{b} \times \vec{c})$ **(05 Marks)**

OR

2   a.   Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\dfrac{\theta}{2}\cdot\cos\dfrac{n\theta}{2}$.   **(06 Marks)**

    b.   Find the sine of angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$ **(05 Marks)**

    c.   Find the value of λ, so that the vector $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are coplanar.   **(05 Marks)**

Module-2

3   a.   If $y = \tan^{-1}x$, prove that
$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$ **(06 Marks)**

    b.   Find the angle between the radius vector and tangent to the curve $r = a(1 - \cos\theta)$ **(05 Marks)**

    c.   If $u = \sin^{-1}\left(\dfrac{x^3 + y^3}{x + y}\right)$ prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\tan u$.   **(05 Marks)**

OR

4   a.   Find the pedal equation of the curve $r = 2(1 + \cos\theta)$ **(06 Marks)**

    b.   Find the total derivative of $u = x^3y^2$, where $x = e^t$, $y = \log t$.   **(05 Marks)**

    c.   Obtain the Maclaurin's series expansion of the function $\sin x$.   **(05 Marks)**

Module-3

5   a.   Evaluate $\displaystyle\int_0^\pi x\cos^6 x\, dx$ **(06 Marks)**

    b.   Evaluate $\displaystyle\int_0^1\int_0^3 x^3 y^3\, dx\, dy$ **(05 Marks)**

    c.   Evaluate $\displaystyle\int_0^1\int_0^1\int_0^1 (x + y + z)\, dx\, dy\, dz$ **(05 Marks)**

OR

6 a. Evaluate $\int\limits_{0}^{\pi/2} \sin^6 x \cos^5 x \, dx$ using Reduction formula. **(06 Marks)**

b. Evaluate $\int\limits_{0}^{1}\int\limits_{x}^{\sqrt{x}} xy \, dy \, dx$ **(05 Marks)**

c. Evaluate $\int\limits_{0}^{1}\int\limits_{0}^{1}\int\limits_{0}^{y} xyz \, dx \, dy \, dz$ **(05 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$. Determine the velocity and acceleration at t = 2. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$. **(05 Marks)**

c. Find the constants a and b, such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. **(05 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at t = 1 and t = 2. **(06 Marks)**

b. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$ **(05 Marks)**

c. Find 'a' for which $\vec{F} = (x + 3y)i + (y - 2z)j + (x + az)k$ is solenoidal. **(05 Marks)**

Module-5

9 a. Solve $\dfrac{dy}{dx} = 1 + \dfrac{y}{x} + \left(\dfrac{y}{x}\right)^2$ **(06 Marks)**

b. Solve $x^2 \dfrac{dy}{dx} = 3x^2 - 2xy + 1$ **(05 Marks)**

c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$ **(05 Marks)**

OR

10 a. Solve $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ **(06 Marks)**

b. Solve $x \dfrac{dy}{dx} = y + x \cos^2\left(\dfrac{y}{x}\right)$ **(05 Marks)**

c. Solve $(x^4 + y^2)dy = 4x^3y \, dx$ **(05 Marks)**

* * * * *